Physics 218B **Plasma Physics** **Winter 2010**

Problem Set II: Due Wednesday, February 17

1.) Kulsrud 5.2

2.) Kulsrud 5.3

3.) Kulsrud 5.4

4.) Kulsrud 4.1 (skip second paragraph)

5.) Kulsrud 7.4

6a.) i.) Revisit the Sweet-Parker reconnection problem, discussed in class. Now, consider a very weakly collisional plasma, so that *electron viscosity* is the dominant dissipation coefficient, i.e. now

$$\frac{\partial \underline{\mathrm{B}}}{\partial t} = \nabla \times \underline{\mathrm{v}} \times \underline{B} + \eta \nabla^2 \underline{B} - \mu \nabla^2 \nabla^2 \underline{B} ,$$

where μ has dimensions of $\ell^2{}_*$ diffusion. Hereafter, neglect η. Following the analysis in class, calculate the thickness of the reconnection layer and the reconnection velocity. Express your answer in terms of an effective "So-and-So Number". Discuss your result as compared to the classic Sweet-Parker result.

ii.) What would be the effect of finite fluid viscosity on the Sweet-Parker process?

b.) Summarize Section 14.3 of Kulsrud. Specifically comment on what the Uzdensky Model adds to the discussion of Sweet-Parker reconnection as discussed in class.

7a.) Show that for incompressible MHD in two dimensions, the basic equations can be written as:

$$(\partial_t + \underline{v} \cdot \nabla)\nabla^2 \phi = (B \cdot \nabla)\nabla^2 A + \nu \nabla^2 \nabla^2 \phi + \tilde{f}$$

$$(\partial_t + \underline{v} \cdot \nabla)A = \eta \nabla^2 A.$$

Here ν is viscosity, η is resistivity, $\underline{v} = \underline{\nabla}\phi \times \hat{z}$ and $\underline{B} = \underline{\nabla}A \times \hat{z}$. $\tilde{f}$ is a random force. Take $P = P(\rho)$.

b.) Take $\underline{B} = B_0 \hat{x}$ to be a weak in-plane magnetic field. Calculate the real frequency and damping for Alfven waves.

c.) Using quasilinear theory, calculate the turbulent resistivity induced by a spectrum of Alfven waves in 2D MHD. For $\nu \to 0$, interpret your result in terms of the freezing-in-law. Why does viscosity enter your result for part i.)? Why does η enter? Contrast these.

d.) Taking $\underline{B} = B_0 \hat{x}$ and $\langle \tilde{V}_y \tilde{A} \rangle = -\eta_T \partial A_0 / \partial y$ as a definition of turbulent resistivity η_T. Show that at stationarity

$$\eta_T = \eta \langle \tilde{B}^2 \rangle / B_0^2,$$

assuming the system has periodic boundary conditions. Discuss your result and its implications. This is a famous result, referred to as the Zeldovich Theorem, after Ya.B. Zeldovich.

8a.) Use quasi-linear theory to write a mean field electron momentum equation for current-driven ion acoustic waves. Include weak collisions.

b.) What does your result imply about Sweet-Parker type reconnection at low collisionality? In your answer, treat the electron distribution self-consistently. Give the anomalous resistivity explicitly, in terms of the wave spectrum.

c.) Use the approach in Section 14.7 of Kulsrud to quantify how far above marginality the CDIA can be expected to get. Express your answer in terms of the parameters of the problem. Clarify the momentum transfer process and the ultimate fate of the momentum.

9.) Kulsrud 4.4

10.) Kulsrud 4.5